DeepSphere: towards an equivariant graph-based spherical CNN

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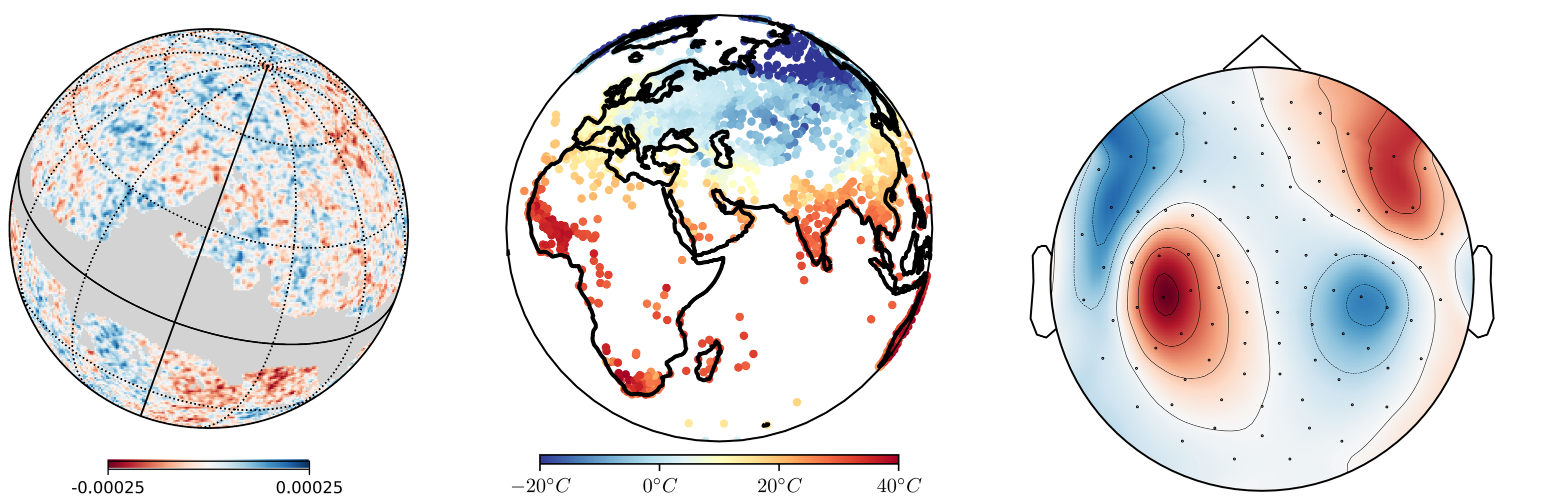
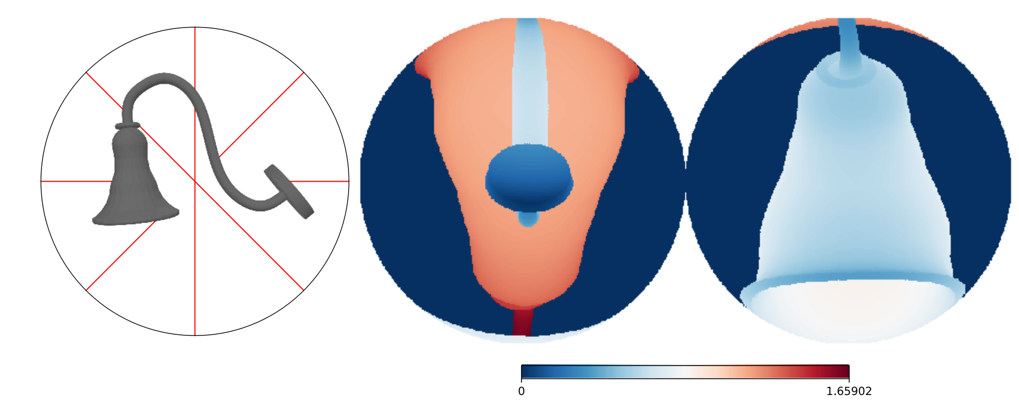
We have spherical data. How can we use a neural network with them?



Intrinsically spherical data:
- cosmic microwave background
- daily temperature
- brain activity (MEG)

Project the data on the sphere to exploit the **rotational symmetry** of any task.

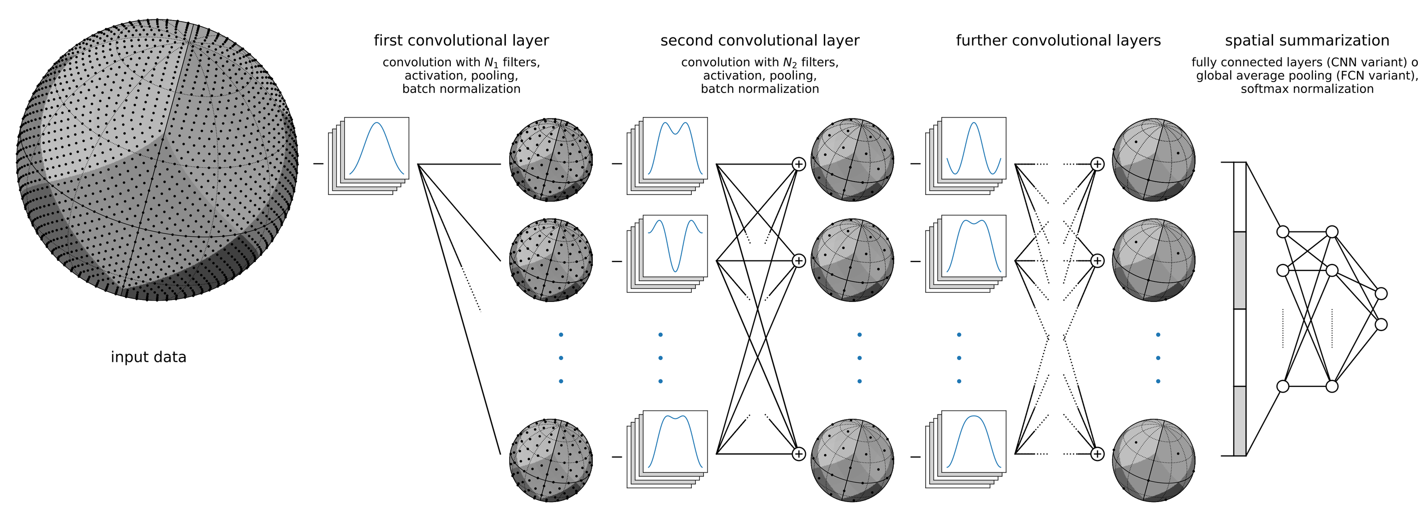


Projection of a 3D shape from SHREC-17.

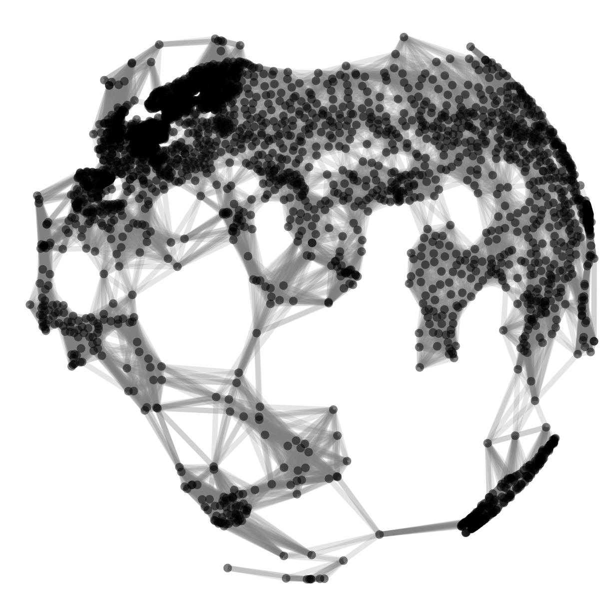
DeepSphere:

1. Model the sampled sphere as a graph.
2. Use a Laplacian-based graph neural network.

=> **Efficient** and **equivariant** spherical CNN.



Bonus: **flexible sampling**



Graph between weather stations.

Different graphs lead to different symmetries.
- **Geometric graphs**: translations and rotations.
- **General graphs**: node permutation.

Why is your graph convolution spherical and equivariant?

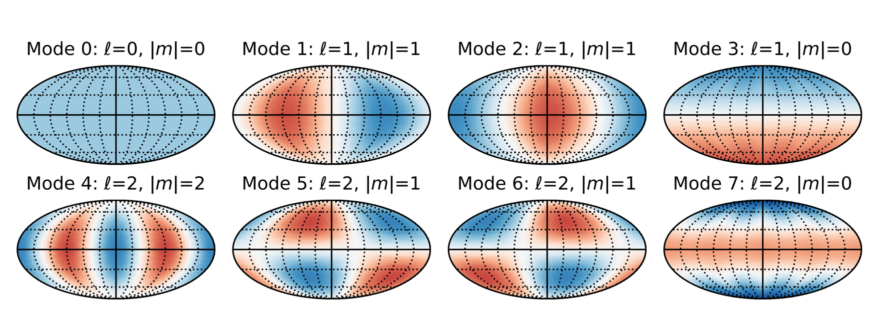
Observation: the graph Laplacian's eigenvectors are close to the spherical harmonics.

Reasoning:

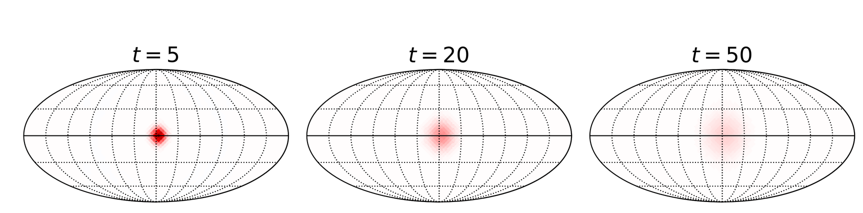
1. The graph Fourier transform is similar to the spherical harmonic transform.
2. Convolution is a multiplication in the spectral domain.
3. The graph convolution is close to the spherical convolution.

Consequence: graph convolution is (almost) rotation equivariant.

Spatial properties of graph filters:
- Invariant to localization => equivariance to SO(3) rotations
- Isotropic kernel

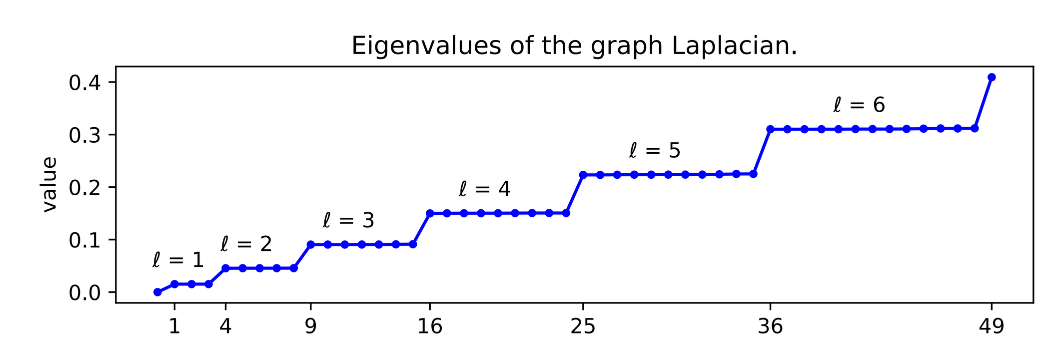


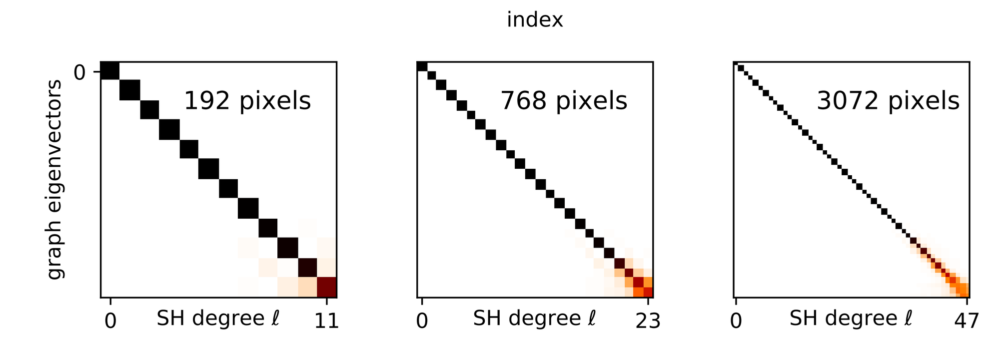
Graphs eigenfunctions.



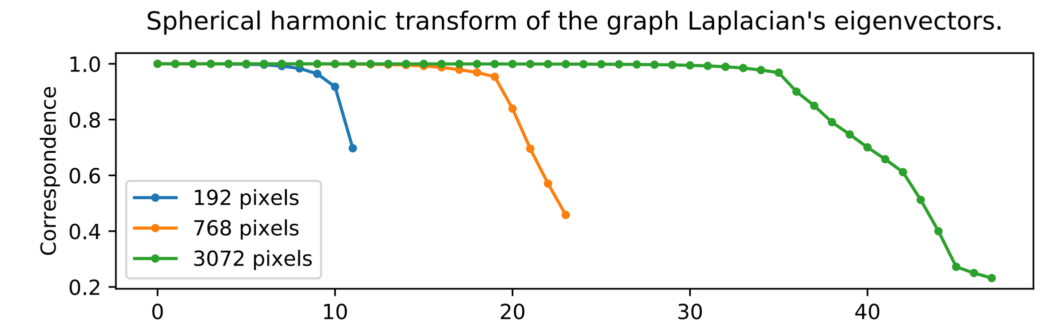
Example graph filter (heat kernel).

DeepSphere v2 (coming soon):
- Empirical correspondence of the eigenspaces.
- Proof of convergence.



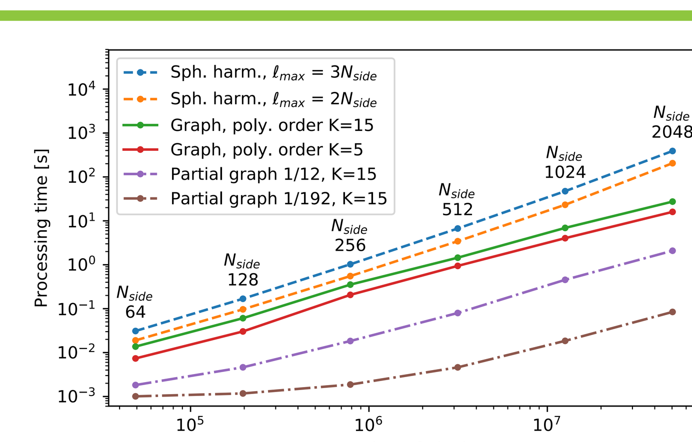


Spherical harmonic transform of the graph Laplacian's eigenvectors.



Then, how is it different from spherical convolution? (used in [Cohen] and [Esteves])

(=) Equivariant to rotations (almost).
(++) Fast: $\mathcal{O}(N)$ vs $\mathcal{O}(N^{3/2})$.
(++) Flexible: accommodates any sampling and partial observations.
(+) Easy to implement (use general & efficient graph NN implementations).
(?/-) Invariant instead of equivariant to the 3rd rotation (isotropic filters). Graph NNs only do same-equivariance and invariance.

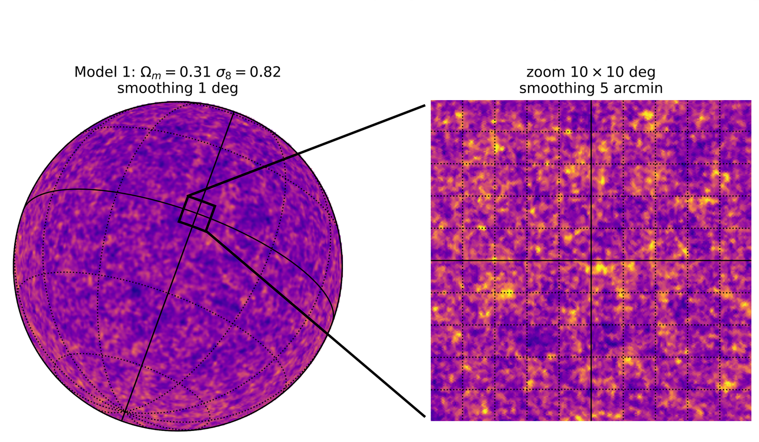


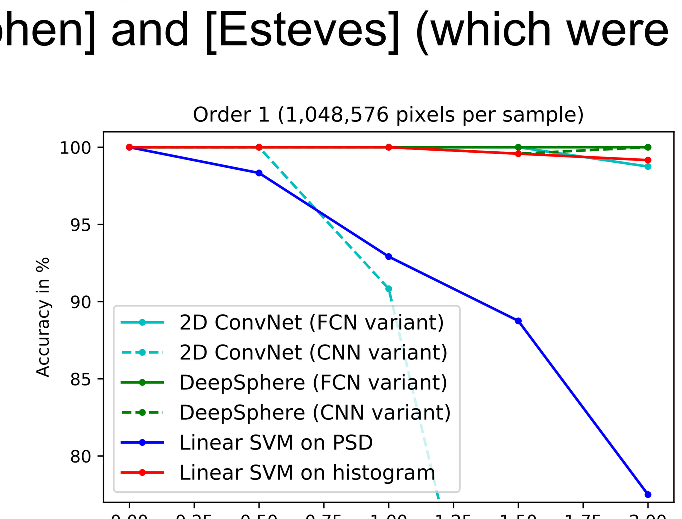
You pay for what you use on irregular samplings, but equivariance needs investigation.

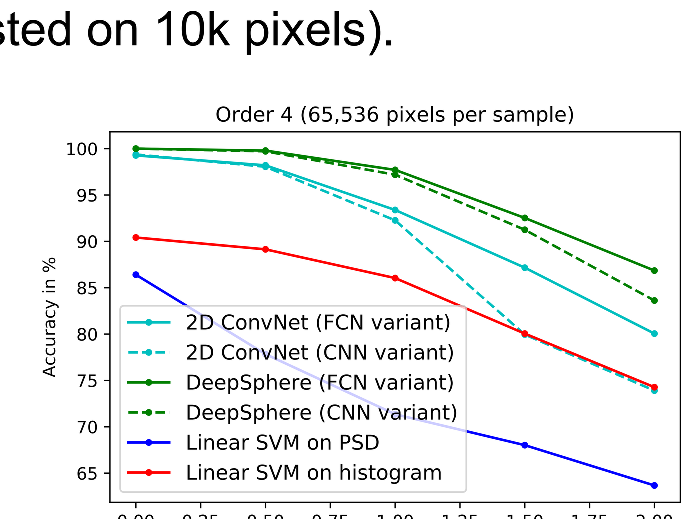
Show me some results!

Task: Discriminate against cosmological models. The goal is to identify the model that best fits our observations of the universe.

Result: DeepSphere beats ConvNet on 2D projections and SVM baselines. Too many pixels (12M) for [Cohen] and [Esteves] (which were tested on 10k pixels).







Recognition of 3D shapes (SHREC-17):
- Same accuracy as [Cohen] and [Esteves].
- Computationally much more efficient.
- Less parameters.

=> Equivariance to 3rd rotation is an unnecessary price to pay.

	performance		size	speed	
	F1	mAP	params	inference	training
$SO(3)$ [Cohen et al.]	-	0.676	1400 k	19.0 ms	50 h
S^2 [Esteves et al.]	79.36	0.685	500 k	9.8 ms	3 h
graph [DeepSphere]	80.65	0.686	190 k	1.6 ms	40 m

Github

https://github.com/SwissDataScienceCenter/DeepSphere

References

- Cohen, Geiger, Köhler, Welling, Spherical CNNs, 2018.
- Esteves, Allen-Blanchette, Makadia, Daniilidis, Learning SO(3) equivariant representations with spherical CNNs, 2018.
- Perraudin, Defferrard, Kacprzak, Sgier, Deepsphere: Efficient spherical convolutional neural network with healpix sampling for cosmological applications, 2018.